

**Exercise 19**

Find the derivative of the function.

$$h(t) = (t + 1)^{2/3}(2t^2 - 1)^3$$

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**Solution**

Take the derivative using the product rule and the chain rule.

$$\begin{aligned} h'(t) &= \frac{dh}{dt} = \frac{d}{dt}[(t + 1)^{2/3}(2t^2 - 1)^3] \\ &= \left[ \frac{d}{dt}(t + 1)^{2/3} \right] (2t^2 - 1)^3 + (t + 1)^{2/3} \left[ \frac{d}{dt}(2t^2 - 1)^3 \right] \\ &= \left[ \frac{2}{3}(t + 1)^{-1/3} \cdot \frac{d}{dt}(t + 1) \right] (2t^2 - 1)^3 + (t + 1)^{2/3} \left[ 3(2t^2 - 1)^2 \cdot \frac{d}{dt}(2t^2 - 1) \right] \\ &= \left[ \frac{2}{3}(t + 1)^{-1/3} \cdot (1) \right] (2t^2 - 1)^3 + (t + 1)^{2/3} [3(2t^2 - 1)^2 \cdot (4t)] \\ &= \frac{2}{3}(t + 1)^{-1/3}(2t^2 - 1)^3 + 12t(t + 1)^{2/3}(2t^2 - 1)^2 \\ &= \frac{2}{3}(t + 1)^{-1/3}(2t^2 - 1)^2[(2t^2 - 1) + 18t(t + 1)] \\ &= \frac{2}{3}(t + 1)^{-1/3}(2t^2 - 1)^2(20t^2 + 18t - 1) \end{aligned}$$